© 2026 International Conference on Multidisciplinary Sciences and Innovations . All Rights Reserved.

DYNAMIC MARKET RISK AND NON-SYNCHRONOUS TRADING IN BETA ESTIMATION: EVIDENCE FROM THE UZBEKISTAN STOCK MARKET

Rakhimov Abdukhalil Toshbotirovich
Researcher, Tashkent State University of Economics, Tashkent, Uzbekistan
Senior teacher, Department of Digital Technologies, Perfect-University

Abstract. This paper examines whether the systematic risk of stocks listed on the Republican Stock Exchange "Toshkent" is stable over time and whether conventional CAPM betas are distorted by non-synchronous trading. Using daily excess returns of four actively traded issuers – Hamkorbank (HMKB), Qizilqumsement (QZSM), Uzsanoatqurilishbank (SQB) and Uzmetkombinat (UZMK) – against the UCI index over 4 July 2022 – 30 June 2026, rolling CAPM betas are estimated with 60-, 120- and 180-observation windows, parameter stability is tested with a beta–time interaction regression and the supF test of Andrews (1993), and OLS betas are compared with the Dimson (1979) and Scholes–Williams (1977) estimators. Rolling betas display substantial temporal variation: the beta of UZMK ranges from −0.45 to 1.28, and SQB exhibits a significant upward trend ($p = 0.036$), although no discrete structural break is detected. Full-sample OLS betas are close to zero and insignificant, and the market factor explains less than 2% of daily return variation. Dimson and Scholes–Williams corrections raise betas by up to 0.09 points for SQB and UZMK – the stocks with the highest share of zero-return days – yet lagged and leading market returns are never jointly significant. The evidence supports time-varying beta but offers only weak support for a systematic non-synchronous-trading bias.

Keywords: CAPM; systematic risk; rolling beta; beta stability; non-synchronous trading; Dimson beta; Scholes–Williams beta; zero-return days; frontier markets; Uzbekistan.

JEL classification: G12; G14; G15; C22; C58.

1. Introduction

The beta coefficient of the CAPM of Sharpe (1964) and Lintner (1965) remains the most widely used summary measure of systematic risk in valuation and portfolio management. Its usefulness rests on two conditions that are rarely examined jointly in frontier markets. First, beta is treated as a constant parameter, whereas evidence accumulated since Blume (1971) shows that individual-stock betas drift, cluster in episodes and occasionally reverse sign. Second, OLS estimation presumes that stock and index returns are recorded synchronously; when a share trades infrequently, its observed return incorporates market information with a delay and the OLS beta is biased (Scholes and Williams, 1977; Dimson, 1979). Both conditions are questionable in Uzbekistan, where trading on the Republican Stock Exchange "Toshkent" (UZSE) is concentrated in a small number of issuers, daily turnover is thin and a sizeable share of trading days ends without any price change. Nevertheless, practitioners routinely apply full-sample or imported betas without evidence that they are either stable or free from microstructure bias.

This paper combines two strands of analysis in a single design. In the first stage, rolling CAPM betas are estimated for four of the most actively traded issuers over July 2022 – June 2026 and subjected to two formal stability tests. In the second stage, we ask whether the observed variation could be a mechanical artefact of non-synchronous trading: betas are re-estimated with the Dimson (1979) and Scholes–Williams (1977) estimators, the joint significance of lagged and leading market returns is tested, and the size of the corrections is related to the share of zero-return days. Our results show that betas vary materially over time and that corrections are economically visible for the least actively traded stocks, but that the formal lead–lag evidence is weak.

2. Literature review

© 2026 International Conference on Multidisciplinary Sciences and Innovations . All Rights Reserved.

Early tests of beta stability (Blume, 1971) established that individual-stock betas are considerably less stable than portfolio betas and are often better described as random coefficients than as constants. In emerging markets the case for time variation is stronger still: Harvey (1995) documents that returns are more predictable and less integrated with world markets, and Bekaert and Harvey (1995) model integration as a regime-switching process. Rolling-window OLS remains the most transparent way of visualising such variation, complemented by tests of parameter instability with an unknown change point (Andrews, 1993; Hansen, 1997). On measurement, Scholes and Williams (1977) show that OLS betas of thinly traded stocks are biased downward and propose a consistent estimator combining lagged, contemporaneous and leading slopes; Dimson (1979) offers a simpler aggregated-coefficients estimator; Lo and MacKinlay (1990) provide the general treatment. The share of zero-return days is a standard proxy for trading frictions, particularly informative in emerging markets where volume data are unreliable (Bekaert, Harvey and Lundblad, 2007). As Fama and French (2004) caution, empirical failures of the CAPM may reflect either model misspecification or measurement problems; separating the two is the purpose of the design adopted here.

3. Data and methodology

The analysis uses daily closing prices of HMKB, QZSM, SQB and UZMK and the UZSE composite index UCI over 4 July 2022 – 30 June 2026, covering the banking sector (HMKB, SQB) and the real sector (QZSM – cement; UZMK – ferrous metallurgy). After removal of duplicate ticker–date records and alignment with index trading days, 917 (HMKB) to 928 (UZMK) daily observations per stock are available. Excess returns are simple daily returns minus a constant risk-free rate of 0.0700% per trading day; since a constant subtracted from both sides is absorbed by the intercept, slope estimates and test statistics are unaffected. Mean daily excess returns range from −0.127% (UZMK) to 0.200% (HMKB), with standard deviations of 2.59% to 4.63%. For each stock the excess-return market model $R_{it} - r_{ft} = \alpha_i + \beta_i(R_{mt} - r_{ft}) + \varepsilon_{it}$ is estimated on rolling windows of W = 120 consecutive observations (approximately six months), with 60 and 180 observations as robustness windows; each estimate is reported with its 95% confidence interval, window R^2 and p-value.

Because visual variation in a rolling path is subject to sampling error, two formal tests are applied to the full sample. The first interacts the market excess return with a scaled time index $\tau_t = t/T \in [0, 1]$, so that the coefficient δ on the interaction measures the total change in beta between the beginning and the end of the sample (H_0: $\delta = 0$). The second is the supF test of Andrews (1993) for a single break at an unknown date, computed over the trimmed interval [0.15, 0.85] with Hansen (1997) p-values using the strucchange package. The first has power against gradual drift, the second against a one-off shift. Non-synchronous trading is addressed by the Dimson estimator, in which the stock's excess return is regressed on lagged, contemporaneous and leading market excess returns (k = 2 leads and lags; k = 1, 3 as robustness) and beta is the sum of the 2k + 1 slopes, and by the Scholes–Williams estimator, which rescales the sum of three simple-regression slopes by $1 + 2\rho_m$, where ρ_m is the first-order autocorrelation of the index return. A joint F-test of the lead and lag coefficients provides the statistical evidence of a lead–lag effect, whereas the difference between corrected and OLS betas provides the economic evidence; the two need not coincide. The share of zero-return days serves as a microstructure indicator of trading inactivity. Three hypotheses are tested. H1: betas exhibit economically meaningful time variation. H2: non-synchronous-trading adjustments differ materially from OLS betas. H3: lagged and leading market returns are jointly significant.

4. Empirical results

Table 1 summarises the 120-observation rolling betas and the stability tests. Average market sensitivity is low for all four stocks: mean rolling betas range from 0.062 (QZSM) to 0.234 (HMKB), and medians are lower still. The paths, however, are far from constant. UZMK shows the widest

© 2026 International Conference on Multidisciplinary Sciences and Innovations . All Rights Reserved.

amplitude, from −0.447 to 1.280, and is the only stock whose beta exceeded unity – in 7.5% of windows, in a continuous episode from mid-December 2024 to late April 2025 peaking on 14 March 2025. SQB reached 0.881 in February 2025 and QZSM 0.577 in March 2025, so the first half of 2025 constitutes a common high-beta episode; in the second half of 2025 all three betas reversed sign, with troughs of −0.335 to −0.524 in October–November. Negative betas occur in 15.5% (HMKB) to 30.9% (QZSM) of windows, but most rolling betas are statistically indistinguishable from zero: only 1.1% to 13.1% of window estimates are significant at 5%, and mean window R^2 lies between 0.4% and 1.5%. These conclusions do not depend on the window: mean betas are almost identical across 60, 120 and 180 observations and the ranking of stocks is preserved.

The formal tests qualify the visual impression. The beta–time interaction is positive for all four stocks but significant only for SQB, whose implied beta rises by 0.340 points over the sample ($p = 0.036$). The supF test does not reject parameter stability for any stock, with statistics between 1.54 (QZSM) and 7.97 (SQB, $p = 0.207$). These results are not contradictory: the rolling paths are dominated by temporary episodes – a rise in the first half of 2025 followed by a reversal – rather than by a single permanent shift, against which supF has little power. The evidence is best summarised as economically meaningful but largely episodic time variation, with a statistically detectable upward drift only for SQB, so that H1 is partially supported.

Table 1. Rolling CAPM betas (W = 120) and formal stability tests, July 2022 – June 2026

Ticker	Mean β	Median β	Min β	Max β	SD β	β < 0, %	β sig. 5%, %	Trend δ	p (δ)	supF	p (supF)
HMKB	0.234	0.222	−0.319	0.746	0.220	15.5	3.6	0.213	0.291	7.156	0.279
UZMK	0.157	0.081	−0.447	1.280	0.380	26.9	13.1	0.129	0.251	7.735	0.226
SQB	0.122	0.083	−0.335	0.881	0.241	29.7	7.5	0.340	0.036	7.970	0.207
QZSM	0.062	0.037	−0.524	0.577	0.180	30.9	1.1	0.012	0.942	1.537	1.000

Note. 798–809 rolling estimates per stock. δ is the beta–time interaction coefficient (total change in beta over the sample); supF is computed over [0.15, 0.85] with Hansen (1997) p-values. Source: author's estimates from UZSE price data and the UCI index.

Table 2 compares the OLS, Dimson (±2) and Scholes–Williams estimators on matched samples together with the zero-return diagnostic. Full-sample OLS betas are very small and insignificant, from −0.025 (SQB) to 0.036 (HMKB), with R^2 below 0.2% and well below the mean rolling betas – a consequence of the variance weighting inherent in OLS, since the full-sample slope is dominated by the high-variance episode of late 2022 – early 2023, when rolling betas were close to zero. The Dimson estimator raises the beta of SQB from −0.025 to 0.065 (+0.091 beta points) and that of UZMK from 0.032 to 0.085 (+0.054); the correction is small for QZSM (+0.010) and negative for HMKB (−0.024). Scholes–Williams corrections are of a similar order: +0.053 for UZMK, +0.043 for SQB, +0.039 for QZSM and +0.003 for HMKB. Two qualifications apply. The standard errors of the Dimson betas (0.070–0.125) are two to three times those of OLS, so every Dimson confidence interval contains zero. Moreover, the first-order autocorrelation of the index return is negative (−0.071 to −0.075); since stale prices in index constituents would induce positive autocorrelation, the negative value points to short-run reversals rather than staleness.

Table 2. OLS, Dimson (±2) and Scholes–Williams betas on matched samples and zero-return diagnostic

Ticker	OLS β	Dimson β	Dimson SE	Joint p	SW β	Δ Dimson	Δ SW	Zero-return days	Zero-return share, %	N

© 2026 International Conference on Multidisciplinary Sciences and Innovations . All Rights Reserved.

Ticker	OLS β	Dimson β	Dimson SE	Joint p	SW β	Δ Dimson	Δ SW	Zero-return days	Zero-return share, %	N
SQB	−0.025	0.065	0.102	0.342	0.017	+0.091	+0.043	117	12.68	919
UZMK	0.032	0.085	0.070	0.720	0.084	+0.054	+0.053	92	9.91	924
HMKB	0.036	0.012	0.125	0.309	0.039	−0.024	+0.003	52	5.67	913
QZSM	0.009	0.019	0.107	0.960	0.048	+0.010	+0.039	52	5.62	922

Note. Δ is the correction in beta points relative to the matched OLS beta; joint p is the p-value of the F-test that all lead and lag coefficients are zero; N is the number of matched observations. Scholes–Williams inputs $\rho_m = -0.071$ to -0.075. Source: author's estimates.

The size of a correction must not be equated with statistical evidence of bias. The joint F-test of the lagged and leading coefficients yields p-values of 0.309 (HMKB), 0.960 (QZSM), 0.342 (SQB) and 0.720 (UZMK), so the null of no lead–lag effect is not rejected for any stock. The conclusion is robust to the lag length: across k = 1, 2 and 3 the smallest joint p-value is 0.200 (UZMK, k = 3). The Dimson betas of SQB and UZMK increase with k (to 0.184 and 0.209 at k = 3), and only UZMK at k = 3 (p = 0.014) is individually significant – a result arising from the accumulation of several small positive coefficients rather than from a jointly significant lead–lag block. H3 is therefore not supported, while H2 is supported in the economic sense for SQB and UZMK. The zero-return diagnostic is consistent with the thin-trading hypothesis: SQB (12.7% zero-return days) and UZMK (9.9%) are precisely the stocks with the largest Dimson corrections and the lowest median absolute daily returns (1.2% and 1.1% against 1.9% for HMKB and QZSM). With four securities this association cannot be tested formally and is reported as supporting evidence, not proof of bias.

5. Discussion

Taken together, the two stages yield one coherent conclusion: the CAPM betas of the selected Uzbek stocks vary substantially over time, but this variation cannot be attributed, on the available evidence, to a mechanical non-synchronous-trading effect. Three implications follow. First, for valuation practice, a single full-sample beta is a poor description of market sensitivity: it is dominated by the high-variance period at the start of the sample and understates the sensitivity observed during active episodes. Rolling or recent-window betas, reported with their confidence intervals, are more informative, and the width of those intervals should temper any precise cost-of-capital conclusion. Second, the very low explanatory power of the market factor – below 2% in every specification – indicates that firm-specific, liquidity and structural factors dominate daily price formation, which supports composite risk-assessment frameworks combining market beta with liquidity, leverage and profitability indicators rather than reliance on the CAPM alone. Third, Dimson and Scholes–Williams betas should be reported alongside OLS betas for the least actively traded issuers, while refraining from the claim that OLS betas are systematically biased: the corrections are large in relative terms precisely because the OLS betas are small, and their statistical foundation is weak. Limitations include the four-stock cross-section and four-year sample, and extreme index observations in 2022–2023 that carry disproportionate weight in full-sample regressions. Future work could model the beta path with GARCH or state-space methods, allow for multiple breaks and incorporate liquidity directly into the pricing equation.

6. Conclusion

This paper has examined the systematic risk of four actively traded Uzbek stocks from two complementary angles: the stability of CAPM beta over time and its sensitivity to non-synchronous trading. Rolling estimates reveal pronounced temporal variation, with UZMK reaching a beta of 1.28 in March 2025 and SQB displaying a significant upward trend, whereas full-sample OLS betas are

© 2026 International Conference on Multidisciplinary Sciences and Innovations . All Rights Reserved.

close to zero and the market factor explains only a small share of daily return variation. Dimson and Scholes–Williams adjustments produce economically visible corrections, particularly for SQB and UZMK, the stocks with the highest share of zero-return days; but joint tests for lagged and leading market returns are insignificant for every stock and lag length. The contribution therefore lies in a balanced three-part conclusion: beta in the Uzbek market varies over time; adjusted betas differ materially from OLS betas for the least actively traded issuers; and the formal evidence that this difference reflects a systematic non-synchronous-trading bias is weak. For researchers and practitioners in frontier markets, the results argue for time-varying, interval-based reporting of beta and for supplementing the CAPM with liquidity and firm-specific risk indicators rather than mechanically replacing OLS betas with thin-trading-corrected alternatives.

References

1. Andrews, D. W. K. (1993). Tests for parameter instability and structural change with unknown change point. Econometrica, 61(4), 821–856.
2. Bekaert, G., & Harvey, C. R. (1995). Time-varying world market integration. Journal of Finance, 50(2), 403–444.
3. Bekaert, G., Harvey, C. R., & Lundblad, C. (2007). Liquidity and expected returns: Lessons from emerging markets. Review of Financial Studies, 20(6), 1783–1831.
4. Blume, M. E. (1971). On the assessment of risk. Journal of Finance, 26(1), 1–10.
5. Dimson, E. (1979). Risk measurement when shares are subject to infrequent trading. Journal of Financial Economics, 7(2), 197–226.
6. Fama, E. F., & French, K. R. (2004). The Capital Asset Pricing Model: Theory and evidence. Journal of Economic Perspectives, 18(3), 25–46.
7. Hansen, B. E. (1997). Approximate asymptotic p values for structural-change tests. Journal of Business & Economic Statistics, 15(1), 60–67.
8. Harvey, C. R. (1995). Predictable risk and returns in emerging markets. Review of Financial Studies, 8(3), 773–816.
9. Lo, A. W., & MacKinlay, A. C. (1990). An econometric analysis of nonsynchronous trading. Journal of Econometrics, 45(1–2), 181–211.
10. Scholes, M., & Williams, J. (1977). Estimating betas from nonsynchronous data. Journal of Financial Economics, 5(3), 309–327.
11. Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. Journal of Finance, 19(3), 425–442.